A Conditional Lead-time Distribution for Intermittent Demand Forecasts

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Presentation Overview

- Intermittent Demand in Marketing and Operations
- Formulating a Lead-time Demand Distribution
- Inference Procedure for Lead-time Probability Levels
- Software Implementation (Excel Add-in)

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Intermittent Demand in Marketing and Operations

- **Merchandising Optimization and Revenue Management**
  - accurately model product mix based on future patterns of demand at the item and store level
  - multiple SKUs at multiple locations
- **Inventory Planning**
  - selecting the most appropriate inventory control policy based on type of stocked item
  - grouping of SKUs for individual ABC category

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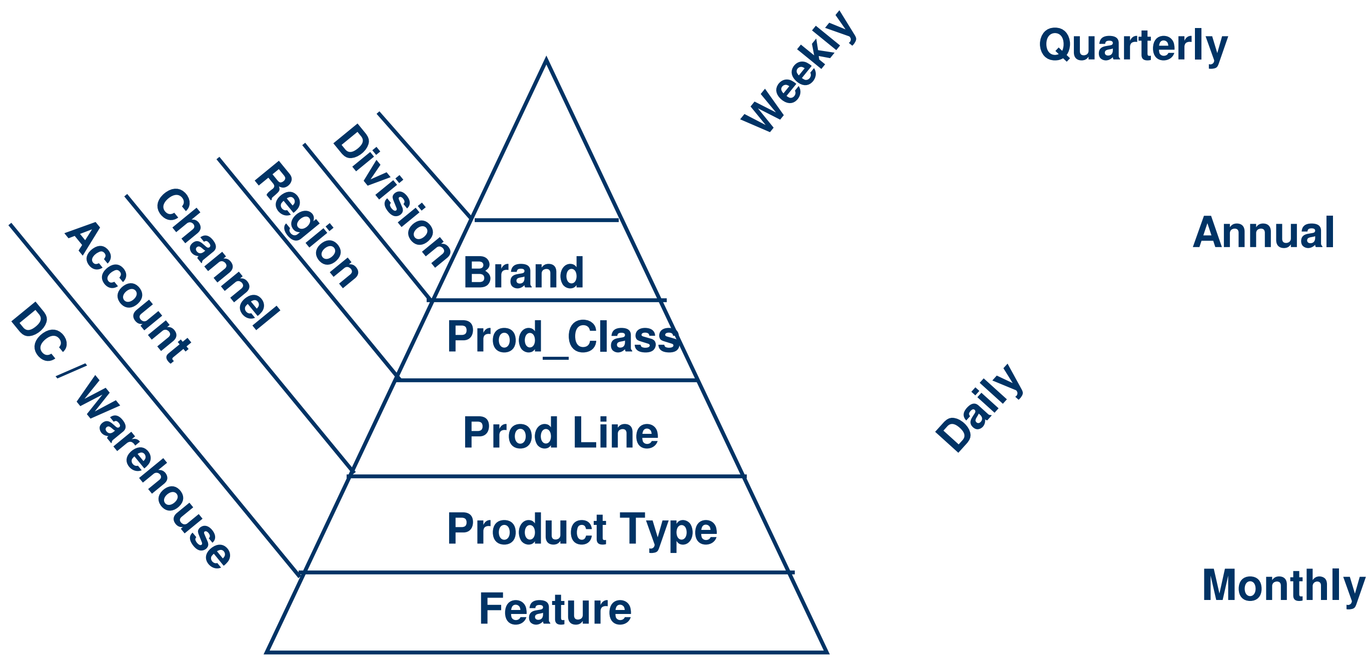
Sources of Homogeneity for Intermittent Demand



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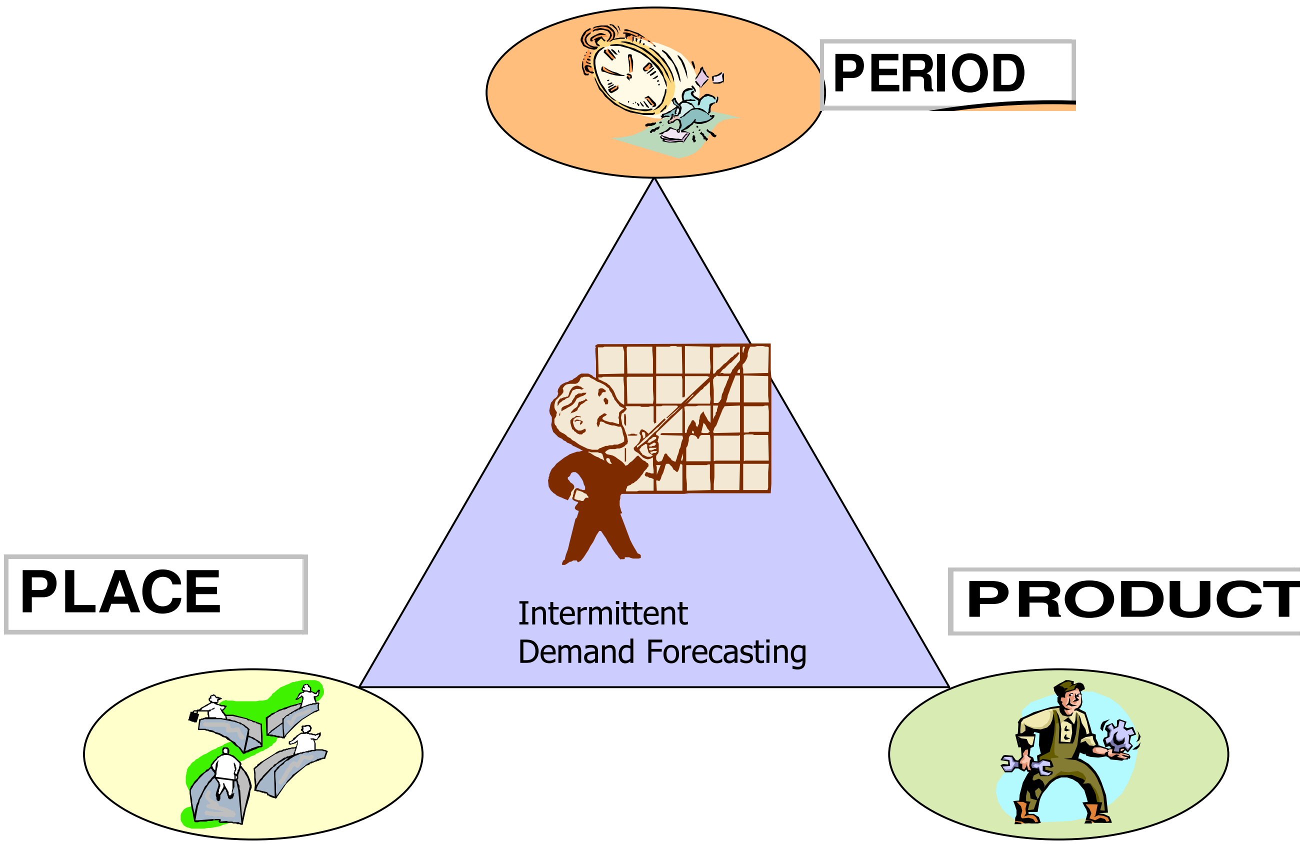
Aligning Product Hierarchy, Customer Segmentation and Time Granularity



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Formulating a Lead-time Demand Distribution

- Select from family of exponential distributions
- Exponential family is closed under location and scale transformations

  $f(x,\theta)$ can be written as $\exp[a(x)+b(\theta)+c(x)d(\theta)]$

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A Model for Lead-Time Demand

A model for Demand Ld over (fixed) lead-time is

$$Ld = \mu + \sigma e ,$$

where

μ is an average level

σ is a scale parameter dependent

e has a random error distribution f(e)de from the exponential family of distributions

Note: Exponential family is location-scale invariant and covers many types of distributions, including the Normal distribution.

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Inference Procedure

- Transform $Ld_i \leftrightarrow \{m(Ld), s(Ld), \mathbf{O_i}\}$ and $e_i \leftrightarrow \{ m(e), s(e), \mathbf{O_i}\}$

  where $\mathbf{O_i} = (Ld_i – m(\underline{Ld}))/ s(\underline{Ld}) = (e_i – m(\underline{e}))/s(\underline{e})$ (i = 1, …, n), and m(.) and s(.) are location and scale statistics, respectively;
- A Reduced Model: Determine the conditional distribution of m(e) and s(e), conditional on observed values of $\mathbf{O_i}$,
- The (conditional) posterior distribution for (μ, σ) is induced from the relationships $m(e) = (m(Ld) - \mu)/ \sigma$, and $s(e) = s(Ld) / \sigma$
- For this posterior distribution, determine a region of constant tail probability

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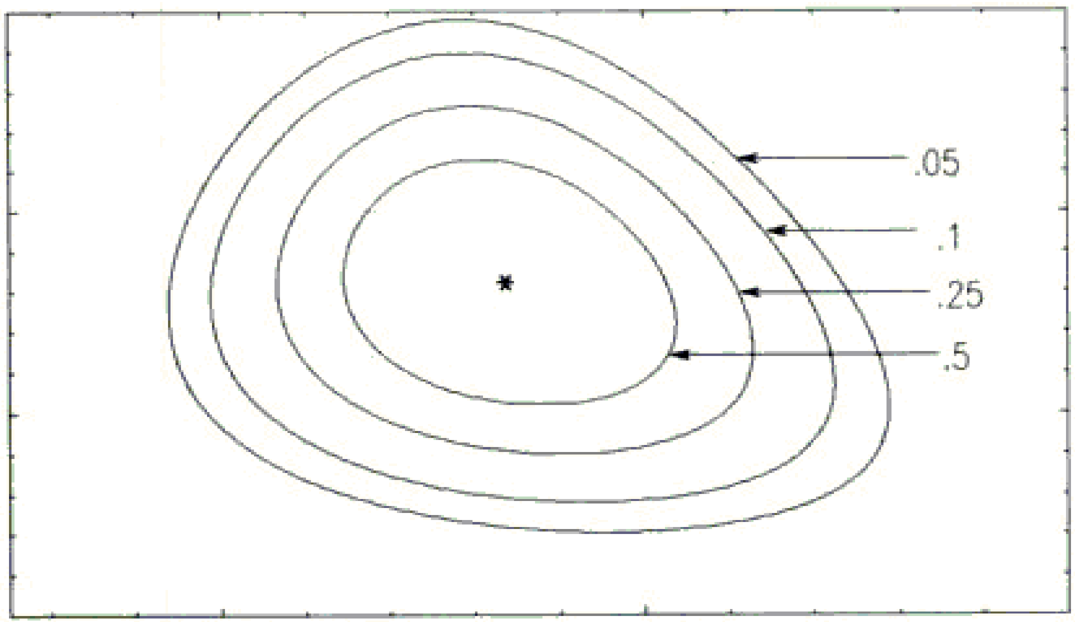
Inference Procedure (cont'd)



Let $\theta = \{\mu,\sigma\}$. Pr_{θ} ($\theta \in$ R | l(x), s(x), **O) = 1 - α**

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Software Implementation (as Excel Add-in)

- Use SSOE models to bootstrap distribution of total lead-time demand
  - non-constant variance
  - constant or variable lead-times
- Use exponential family for total lead-time demand distribution
- Use 'information measure' to assess accuracy
  - departure from normality assumptions
- Benchmark with normal distribution

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Summary

- No explicit assumptions made on Demand Event or Demand Interval distributions
- No explicit assumptions on correlations among Demand Events and Intervals
- Determine upper tail percentile of a lead-time distribution from family of exponential distributions

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Some References

- Fraser (19xx). **Linear Models**, Wiley
- Hyndman, et. al. (2001), IJF, 17, 269-286 – empirical bootstrap with SSOE for generating lead-time demand
- Snyder, et.al. (2002), IJF, 18, 5-18 – parametric bootstrap with SSOE for generating lead-time demand

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